Differential Form of Basic Profile Coupling Equations applied to Operative Elements of Gerotor Pumps

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KEYWORDS

Profile coupling equation, Differential form, Gerotor pumps

ABSTRACT

Interaction conditions of coupled profiles, determined by basic theorem of gearing coupling, can be presented in analytical form. In literature, they are mostly represented graphically. Analytic form is proven especially useful in projecting and research of gearing coupling, it is being used as theoretical basis for non-standard mechanisms for different purposes, at profiling of cutting tools made with envelope method, etc.

The form of profile coupling equations, which is shown in this paper, is applied on definition of gear profile on a gerotor pump, for known profile of the rotor gear (star) of this pump.

1. INTRODUCTION

Interaction conditions of coupled profiles, determined by basic theorem of gearing coupling, can be presented in analytical form. In literature, they are mostly represented graphically. Analytic form is proven especially useful in projecting and research of gearing coupling, it is being used as theoretical basis for non-standard mechanisms for different purposes, at profiling of cutting tools made with envelope method, etc.

2. DIFFERENTIAL FORM OF BASIC PROFILE COUPLING EQUATIONS FOR EXTERNAL GEARING

Let the profile π_1 be given by known equation:

$$\tilde{y}^{(1)} = f_1(x^{(1)})$$

in coordinate system $O_1x^{(1)}y^{(1)}$ (fig. 1).

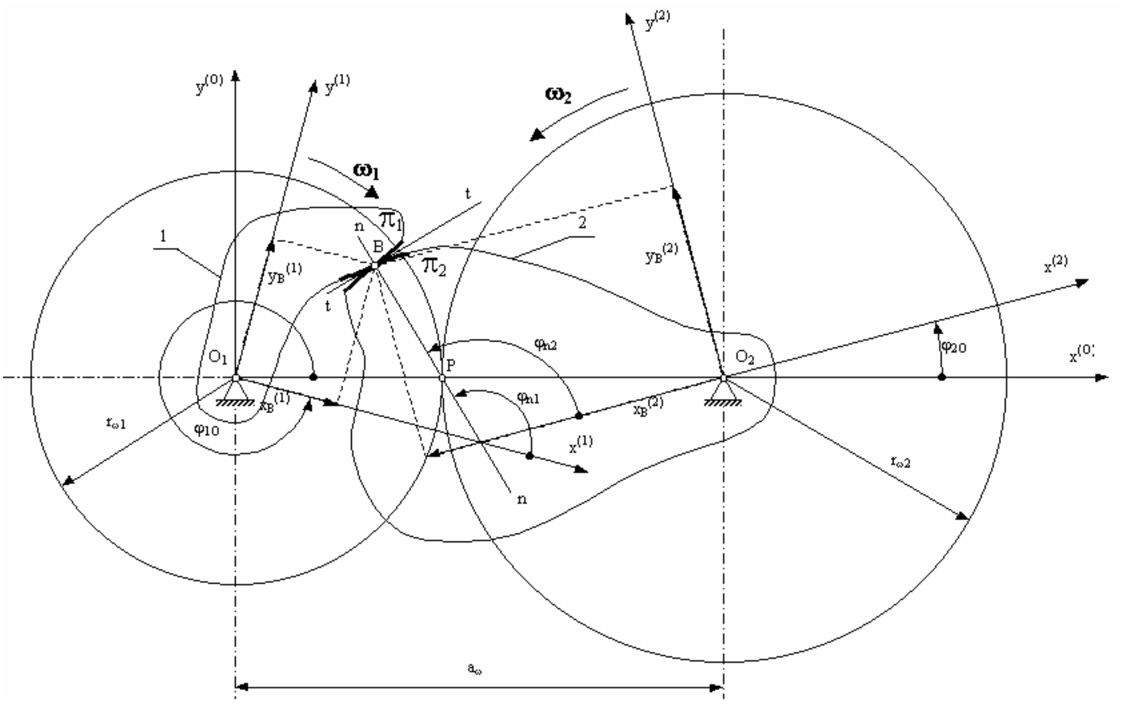

Figure 1: External profile coupling.

Equation of perpendicular line n-n, which passes through the touch point B of coupled profile B, has following form:

$$\left(x^{(1)} - x_B^{(1)}\right) + f'\left(x^{(1)}\right)_B \left(y^{(1)} - y_B^{(1)}\right) = 0\,, \tag{1}$$

where:

$f'\left(x^{(1)}\right)_B = \left(\dfrac{dy^{(1)}}{dx^{(1)}}\right)_B$ - angular coefficient of tangent incline in the touch point B;

$x^{(1)}$, $y^{(1)}$ – current coordinates of the points on perpendicular line;

$x_B^{(1)}, y_B^{(1)}$ - coordinates of the touch points B of the profile in the system $O_1x^{(1)}y^{(1)}$.

According to the basic theorem of gearing coupling, perpendicular line n-n has to pass through the pole P with coordinates:

$$x_P^{(1)} = r_{\omega 1} \cos\varphi_{1O} \qquad y_P^{(1)} = -r_{\omega 1} \sin\varphi_{1O} \tag{2}$$

Equation (1) of the perpendicular line can be written in the following form:

$$tg\varphi_t = \left(\frac{dy^{(1)}}{dx^{(1)}}\right)_B = -\frac{x^{(1)} - x_B^{(1)}}{y^{(1)} - y_B^{(1)}} = -\frac{x_P^{(1)} - x_B^{(1)}}{y_P^{(1)} - y_B^{(1)}} \tag{3}$$

Taking in consideration relations of coordinates of point P, this equation can be written in the following form:

$$\left(x_B^{(1)} - r_{\omega 1} \cos\varphi_{1O}\right) + \left(\frac{dy^{(1)}}{dx^{(1)}}\right)_B \left(y_B^{(1)} + r_{\omega 1} \sin\varphi_{1O}\right) = 0 \tag{4}$$

This equation of the line perpendicular to given profile π_1, which passes through the pole P at the moment of catch of coupled profiles, sometimes is called differential form of gearing equation.

Radius of the centroid $r_{\omega 1}$ can be expressed with interaxial distance a_ω and transmitting ration of the mechanism $u_{21} = \frac{\omega_2}{\omega_1}$, with following relations:

$$\begin{aligned} u_{21} &= \frac{\omega_2}{\omega_1} = -\frac{\overline{PO_1}}{\overline{PO_2}} = -\frac{r_{\omega 1}}{r_{\omega 2}} = \frac{r_{\omega 1}}{a_\omega - r_{\omega 1}} \\ r_{\omega 1} &= a_\omega \frac{u_{21}}{u_{21} + 1} \end{aligned} \tag{5}$$

If we include this relation (5) into expression for differential form of gearing equation, we get:

$$x_B^{(1)} + u_{21}\left(x_B^{(1)} - a_\omega \cos\varphi_{1O}\right) + \left(\frac{dy^{(1)}}{dx^{(1)}}\right)_B \left[y_B^{(1)} + u_{21}\left(y_B^{(1)} + a_\omega \sin\varphi_{1O}\right)\right] = 0 \tag{6}$$

This differential equation of profile coupling enables determination of the angle φ_{1O} for given parameters of the mechanism: interaxial distance a_ω, transmission ratio u_{21} and the equation of one of the profiles π_1 $\left(y^{(1)} = f\left(x^{(1)}\right)\right)$.

Coordinates of the contact points K in fixed coordinate system $O_1x^{(O)}y^{(O)}$, connected with a stand, can be obtained by coordinate transformation by means of following matrix equation:

$$\bar{r}_K^{(O)} = M_{1O}\bar{r}_B^{(1)} \tag{7}$$

where:

$$\bar{r}_K^{(O)} = \begin{bmatrix} x_K^{(O)} \\ y_K^{(O)} \\ 1 \end{bmatrix}; \quad \bar{r}_B^{(1)} = \begin{bmatrix} x_B^{(1)} \\ y_B^{(1)} \\ 1 \end{bmatrix}; \quad M_{1O} = \begin{bmatrix} \cos\varphi_{1O} & -\sin\varphi_{1O} & 0 \\ \sin\varphi_{1O} & \cos\varphi_{1O} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Desired coordinates of points K determine lines of coupling – geometrical collection of contact points:

$$\begin{aligned} x_K^{(O)} &= x_B^{(1)}\cos\varphi_{1O} - y_B^{(1)}\sin\varphi_{1O} \\ y_K^{(O)} &= x_B^{(1)}\sin\varphi_{1O} + y_B^{(1)}\cos\varphi_{1O} \end{aligned} \tag{8}$$

Coordinates of points B on the profile π_2, which is obtained through coupling with given profile π_1 are also determined through coordinate transformation from fixed $O_1x^{(O)}y^{(O)}$ to mobile coordinate system $O_2x^{(2)}y^{(2)}$, through use of following matrix equation:

$$\bar{r}_B^{(2)} = M_{O2}\bar{r}_K^{(O)} \tag{9}$$

where:

$$\bar{r}_B^{(2)} = \begin{bmatrix} x_B^{(2)} \\ y_B^{(2)} \\ 1 \end{bmatrix}; \quad \bar{r}_K^{(O)} = \begin{bmatrix} x_K^{(O)} \\ y_K^{(O)} \\ 1 \end{bmatrix}; \quad M_{O2} = \begin{bmatrix} \cos\varphi_{2O} & -\sin\varphi_{2O} & -a_\omega\cos\varphi_{2O} \\ \sin\varphi_{2O} & \cos\varphi_{2O} & a_\omega\sin\varphi_{2O} \\ 0 & 0 & 1 \end{bmatrix}$$

In final form we obtain:

$$\begin{aligned} x_B^{(2)} &= x_K^{(O)}\cos\varphi_{2O} + y_K^{(O)}\sin\varphi_{2O} - a_\omega\cos\varphi_{2O} \\ y_B^{(2)} &= -x_K^{(O)}\sin\varphi_{2O} + y_K^{(O)}\cos\varphi_{2O} + a_\omega\sin\varphi_{2O} \end{aligned} \tag{10}$$

In the last expression, angle φ_{2O} is determined through the expression:

$$\varphi_{2O} = \int_0^{\varphi_{1O}} u_{21}d\varphi_{1O}$$

If we have constant transmission ratio of the mechanism u_{21} this expression is reduced to a special case:

$$\varphi_{2O} = u_{21}\varphi_{1O}\,. \tag{11}$$

3. DIFFERENTIAL FORM OF BASIC PROFILE COUPLING EQUATIONS FOR INTERNAL GEARING

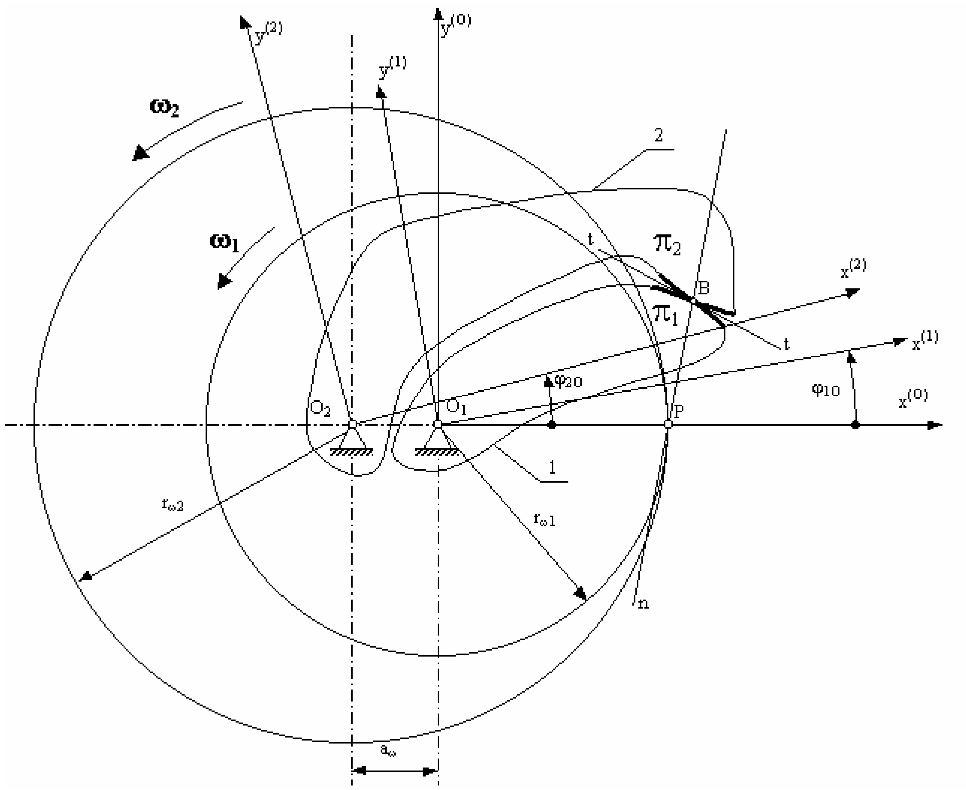

Figure 2: Internal profile coupling.

Expression (5) for the radius of the centroid $r_{\omega 1}$ for internal gearing, according to figure 2 is:

$$r_{\omega 1} = a_\omega \frac{u_{21}}{1 - u_{21}} \tag{12}$$

If we include relation (12) into expression for differential form of gearing equation (4), we get:

$$x_B^{(1)} - u_{21}\left(x_B^{(1)} + a_\omega \cos\varphi_{1O}\right) + \left(\frac{dy^{(1)}}{dx^{(1)}}\right)_B \left[y_B^{(1)} - u_{21}\left(y_B^{(1)} - a_\omega \sin\varphi_{1O}\right)\right] = 0 \tag{13}$$

Solution of this differential equation gives the angle φ_{1O} for given parameters of the mechanism (a_ω, u_{21} and $\left(y^{(1)} = f\left(x^{(1)}\right)\right)$, as it is already mentioned for external gearing.

Coordinates of the contact points K in fixed coordinate system $O_1 x^{(O)} y^{(O)}$, can be obtained by using equations (7) and (8).

Coordinates of points B on the profile π_2 in mobile coordinate system $O_2x^{(2)}y^{(2)}$ are determined through use of matrix equation (9) where:

$$M_{O2} = \begin{bmatrix} \cos\varphi_{2O} & -\sin\varphi_{2O} & a_{\omega}\cos\varphi_{2O} \\ \sin\varphi_{2O} & \cos\varphi_{2O} & -a_{\omega}\sin\varphi_{2O} \\ 0 & 0 & 1 \end{bmatrix}$$

In final form we obtain equations (10) for internal gearing:

$$\begin{aligned} x_B^{(2)} &= x_K^{(O)}\cos\varphi_{2O} + y_K^{(O)}\sin\varphi_{2O} + a_{\omega}\cos\varphi_{2O} \\ y_B^{(2)} &= -x_K^{(O)}\sin\varphi_{2O} + y_K^{(O)}\cos\varphi_{2O} - a_{\omega}\sin\varphi_{2O} \end{aligned} \qquad (14)$$

where $\varphi_{2O} = u_{21}\varphi_{1O}$ with fixed transmission ratio of the mechanism u_{21}.

4. APPLICATION OF BASIC PROFILE COUPLING EQUATIONS TO OPERATIVE ELEMENTS OF GEROTOR PUMPS

Cross-section of operating elements of a gerotor pump (1 – rotor/star, 2 – ring/casing) is shown in figure 3.

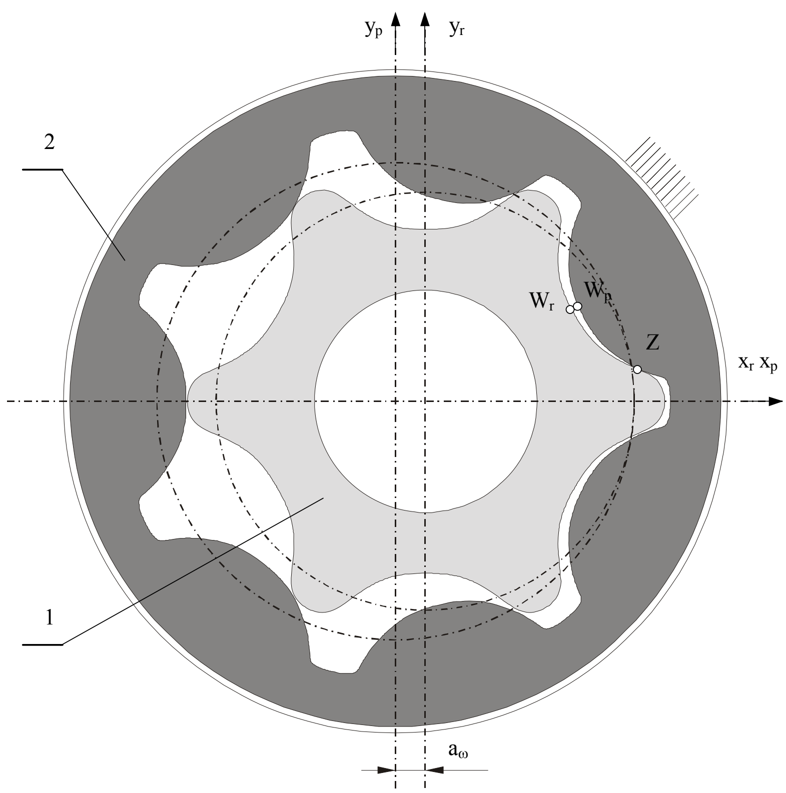

Figure 3: Cross-section of operating elements of a gerotor pump.

We start from the side profile equation $\tilde{y}^{(1)} = f_1(x^{(1)})$ for the rotor gear profile of gerotor pump in the form of complex numbers:

$$z(\beta) = (r_O + r_p)\cdot e^{i\frac{\beta}{z}} + e_1 \cdot e^{i\frac{z+1}{z}\beta} - r_e \cdot e^{i\left[\frac{\beta}{z}+\operatorname{arctg}\left(\frac{\sin\beta}{\frac{r_p}{e_1}+\cos\beta}\right)\right]}$$

or, in paremattic shape:

$$x_r = (r_O + r_p)\cdot \cos\left(\frac{\beta}{z}\right) + e_1 \cdot \cos\left(\frac{z+1}{z}\beta\right) - r_e \cdot \cos\left[\frac{\beta}{z} + \operatorname{arctg}\left(\frac{\sin\beta}{\frac{r_p}{e_1} + \cos\beta}\right)\right]$$

$$y_r = (r_O + r_p)\cdot \sin\left(\frac{\beta}{z}\right) + e_1 \cdot \sin\left(\frac{z+1}{z}\beta\right) - r_e \cdot \sin\left[\frac{\beta}{z} + \operatorname{arctg}\left(\frac{\sin\beta}{\frac{r_p}{e_1} + \cos\beta}\right)\right] \quad (15)$$

where:

r_O – radius of basic fixed circle;

r_p – radius of mobile circle which rolls over fixed circle;

β - angle of rotation of mobile circle over the basic circle;

z – number of gears (segments) on the circumference of basic circle of the rotor $z = \frac{r_O}{r_p}$;

e_1 – excentricity – distance between the point on fixed circle and center of the circle;

r_e – radius of equidistant circle.

Without intention to come into detailed evaluation of geometric parameters of the rotor and the ring of chosen 6/7 gerotor pump, the procedure of obtaining coordinates of the coupling points on the gear profile of this pump's ring, we can evaluate following table, applying expression from chapter 3:

Table 1: Coordinates of the points of gear profile (part 1).

Point No.	**Point caption on fig. 3**	**Coordinates of the points of rotor gear profile [mm]**		**Derivation of expression (15) in touch points B**		$\left(\frac{dy^{(1)}}{dx^{(1)}}\right)_B = \frac{\left(y_r'^{(1)}\right)_B}{\left(x_r'^{(1)}\right)_B}$
		$x_r=x_B^{(1)}$	$y_r=y_B^{(1)}$	$(x'_r{}^{(1)})_B$	$(y'_r{}^{(1)})_B$	
1	Z	24,69878	3,92422			
2		24,49636	4,00591			
3		24,04422	4,09306			
⋮		⋮	⋮			
37		17,56672	10,00051			
38	W	17,50497	10,10650			

Table 1: Coordinates of the points of gear profile (part 2).

Point No.	**Point caption on fig. 3**	**φ_{1O} [O] izraz (13)**	**$x_K^{(O)}$ izraz (8)**	**$y_K^{(O)}$ izraz (8)**	**φ_{2O} [O] izraz (11)**	**Coordinates of the points of ring gear profile [mm]**	
						$x_p=x_B^{(2)}$	**$y_p=y_B^{(2)}$**
1	Z					28,27503	3,98072
2						28,08305	4,06034
3						27,86967	4,13978
⋮						⋮	⋮
37						21,47213	10,27555
38	W					21,44666	10,32817

5. CONCLUSION

Differential form of basic profile coupling equations is the most appropriate form in projecting and research of different ways of profile coupling. It is appropriate for both algorythmisation and for tabular determination of coordinates of the coupled profiles points (using spreadsheet software such as MS Excel), as it is suggested in this paper, on the example of defining coupling profile of the ring for known rotor profile of a 6/7 gerotor pump.

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