

MODE SHAPES OF CENTRIFUGAL PUMP IMPELLER

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ABSTRACT

Problems of vibrating centrifugal pump impellers and likewise structures with complex geometry, are usually being analysed by means of Finite Element Method or experimental methods. The first step in dynamic analysis is to determine natural frequencies and mode shapes. To classify natural frequencies according to vibration mode shapes, theory of circular plates vibration can be used. This paper shows how impeller vibration mode shapes can be classified for one type of centrifugal pump, and it proves that the impeller presented acts very similarly to circular plate.

Keywords: Finite Elements Method, Vibrations, Mode shapes, Impeller

1. INTRODUCTION

Vibration analysis of complex geometric shapes, such as centrifugal pump impeller, is usually performed by means of finite element method. This method is eigenvalue problem in matrix form, therefore the results obtained are coordinates of points in finite element nodes (mode shapes) and list of natural frequencies sorted by size. Identification of these vibration modes requires fair knowledge on analytical solution for similar shapes. Centrifugal pump impeller natural frequencies can be calculated using similar procedure as the one used in plate vibration, since impeller acts similarly to circular plate [2].

2. DYNAMIC EQUATION OF MOTION FOR CIRCULAR PLATE

Thin plate is two-dimensional analogy of Euler-Bernoulli beam (where tangential stresses are neglected). Two-dimensional analogy of Timoshenko beam (including both tangential stresses and inertial forces due to rotation of the plate) is known in literature as Mindlin-Timoshenko theory. Differential equation of free vibration of circular plate in cylindrical coordinates is:

$$D\Delta\Delta w - \rho h \ddot{w} = 0. \quad \dots (1)$$

The goal is to determine conditions where all points on the plate will perform harmonic vibration with same frequency and in the same phase. The shape of function w is assumed:

$$w(r, \varphi, t) = W(r, \varphi, t)e^{i\nu t}. \quad \dots (2)$$

Introducing constant c and λ^2 :

$$c^2 = \frac{D}{\rho h} \quad \lambda^2 = \frac{\omega}{c} \quad \dots (3)$$

After transformations, following differential equation is obtained:

$$\Delta\Delta W - \lambda^4 W = (\Delta - \lambda^2)(\Delta + \lambda^2)W = 0 \quad \dots (4)$$

General solution of equation (4) is:

$$W(r, \varphi) = \frac{\cos}{\sin}(m\varphi) [A\mathfrak{J}_m(\lambda r) + BY_m(\lambda r) + C\mathfrak{J}_m(i\lambda r) + DY_m(i\lambda r)] \quad \dots (5)$$

where $\mathfrak{J}_m(\lambda r)$ and $\mathfrak{J}_m(i\lambda r)$ are first order Bessel functions, and $Y_m(\lambda r)$ and $Y_m(i\lambda r)$ second order Bessel functions. For solid circular plate following solutions are permissible:

$$W(r, \varphi) = \frac{\cos}{\sin}(m\varphi) [A_1\mathfrak{J}_m(\lambda r) + B_1I_m(\lambda r)] \quad \dots (6)$$

For pinned plate (external radius $r = R$) following boundary conditions apply:

$$\begin{aligned} W(R, \varphi) &= 0 \\ \Delta W(R, \varphi) &= 0; \quad \dots (7) \\ \left[\frac{\partial^2 W}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \varphi^2} \right) \right]_{r=R} &= 0 \end{aligned}$$

Introducing (6) into (7), two homogenous equations are derived, and their solution leads to frequency equation.

3. MODE SHAPES OF CIRCULAR PLATES

Vibration mode shapes of circular plates can be represented using nodal lines and nodal circles. Nodal lines and nodal circles consist of stationary points for specific mode shape, and they form geometric patterns known as "Chladni figures". It was proved in [1] that fluid remains attached to white (vibrating) areas of Chladni figures, and it separates from black areas (non-vibrating nodal lines). The shape of these patterns depends only on frequency, and amplitude depends on intensity of turbulence and velocity by which materials moves on the plate.

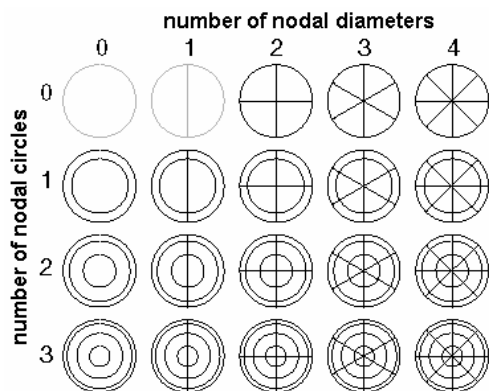


Figure 1. Nodal lines and nodal circles

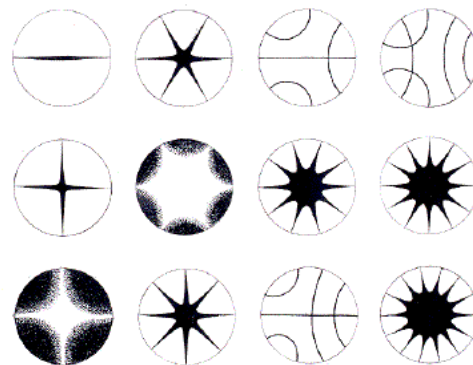


Figure 2. Chladni figures – circular plate mode shapes

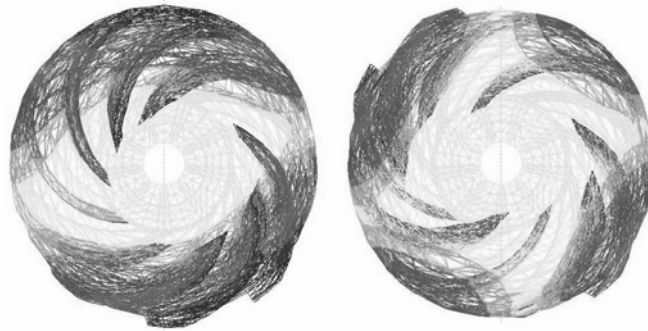


Figure 3. Chladni figures formed by vibrating centrifugal pump impeler

4. CLASSIFICATION OF NATURAL FREQUENCIES DERIVED USING FINITE ELEMENT METHOD BY MEANS OF CHLADNI FIGURES

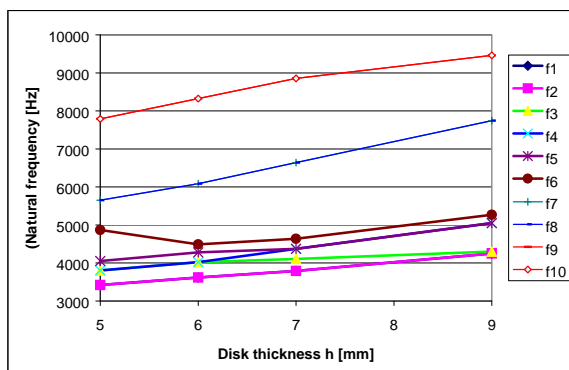
In [3] detailed analysis of influence of constructive parameters to vibration behaviour on one type of centrifugal pump was performed. Natural frequencies were calculated by means of finite element software "I-deas Master Series(c)" for various constructional parameters, such as disk thickness, number of vanes, vane thickness, etc.

Table 1 shows results of calculated natural frequencies for various disk thickness, as they are given by finite element method software. These results are sorted only by size, and there is no distinction between torsional and lateral vibrations. First ten natural frequencies are calculated, and some values are repeated.

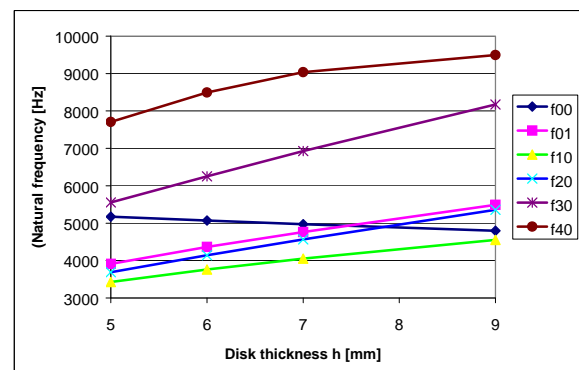
Figure 4.a. shows diagram with false dependence between disk thickness and natural frequencies, since it does not involve vibration classification according to vibration modes. Figure 4.b. shows diagram according to results sorted by vibration modes (right-hand half of table 1). Natural frequencies are not denominated with ordinal numbers, but they have two indexes. First index represents number of nodal diameters, and second one number of nodal circles. Index "00" represents torsional vibration.

Table 1. Natural frequencies calculated using finite element method

Natural frequencies sorted by size					Natural frequencies sorted by vibration modes				
Disk thickness	5 mm	6 mm	7 mm	9 mm	Disk thickness	5 mm	6 mm	7 mm	9 mm
f1	3422	3619	3793	4249	f00	4870	4488	4106	4301
f2	3422	3619	3793	4249	f01	4052	4279	4636	5270
f3	3806	4021	4106	4301	f10	3422	3619	3793	4249
f4	3806	4021	4372	5051	f20	3806	4021	4372	5051
f5	4052	4279	4372	5051	f30	5652	6086	6642	7742
f6	4870	4488	4636	5270	f40	7792	8324	8856	9461
f7	5652	6086	6642	7742					
f8	5652	6086	6642	7742					
f9	7792	8324	8856	9461					
f10	7792	8324	8856	9461					



a) Sorted by size



b) Sorted by vibration mode shapes

Figure 4. Natural frequencies calculated using finite element method

To perform such classification, it is necessary to compare mode shapes of the impeller with Chladni figures of circular plates, identifying nodal diameters and nodal circles. Figure 5 shows results of finite element analysis with appropriate Chladni figures of circular plates.

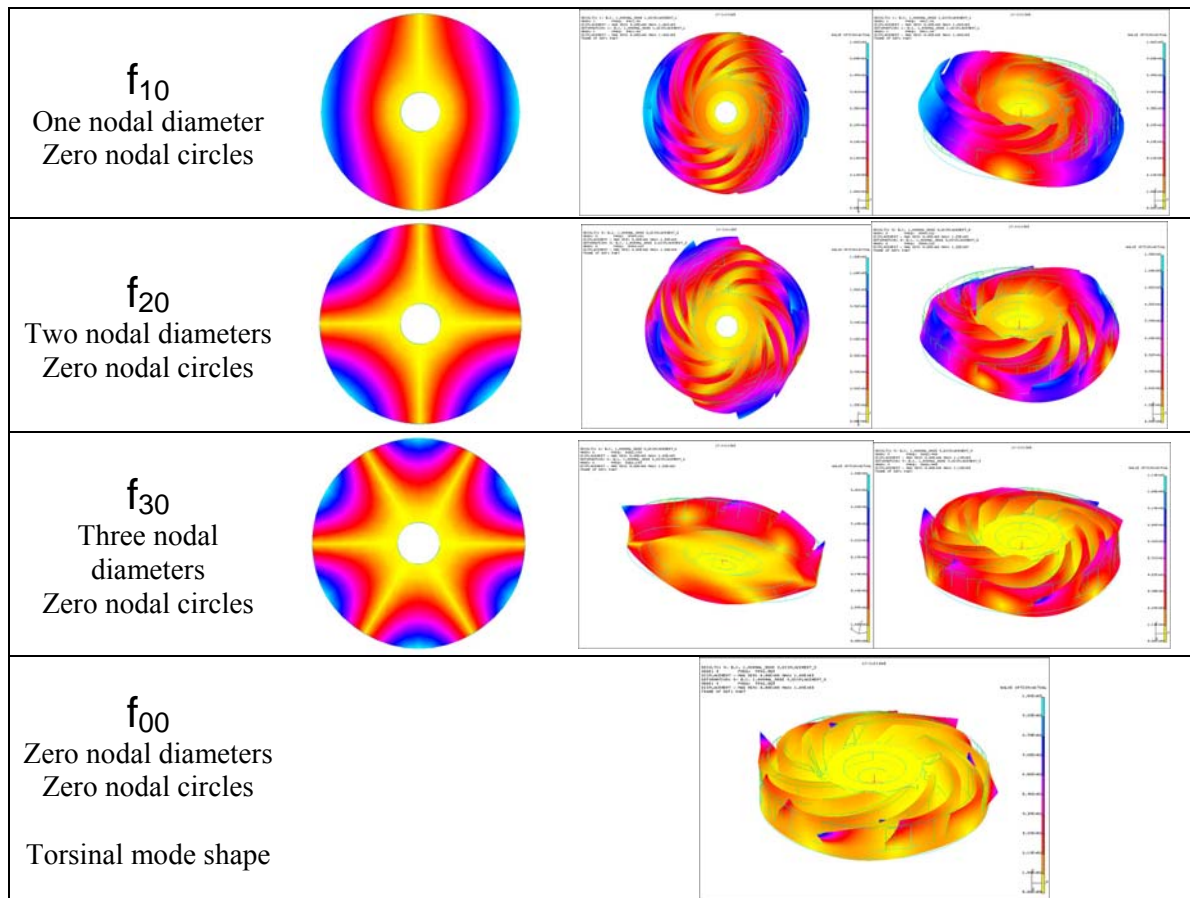


Figure 5. Vibration mode shapes of circular plate and centrifugal pump impeler

5. CONCLUSION

Finite element method gives fair results in calculation of natural frequencies. Nevertheless, if dependence of some parameters is needed, it is necessary to be familiar with physical character of vibrations of structure being analysed. Chladni figures can be of great importance as a basis for identification and classification of vibration modes of plate-shaped structures, such as impellers of some centrifugal pump, rail wheels, plates in computer hard disks, etc.

6. REFERENCES

- [1] Jenny H.: "Kymatik: Wellen und Schwingungen mit ihrer Struktur und Dynamik", Basilius Press, 1967.
- [2] Kushner F.: "Disc Vibration-Rotating Blade and Stationary Vane Interaction", ASME paper No. 79-DET-83, 1979.
- [3] Lemeš S.: "Vibrations of Centrifugal Pumps Used in Automobile Cooling Systems", Master Thesis, University of Sarajevo, 2002
- [4] Lemeš S., Zaimović-Uzunović N., Balić S.: "Identification Of Vibration Mode Shapes In Dynamic Analysis Of A Centrifugal Pump Impeller By Use Of Finite Element Method", *Mašinstvo* 1(6), 15-22, 2002.
- [5] Japikse D., Furst R., Marscher W.D.: "Centrifugal Pump Design and Performance", Wilder, 1997.
- [6] Boulahbal D.: "Self-Excited Vibrations of a Spinning Disk", PhD Thesis, Massachussets Institute of Technology, 1995.