

IDENTIFIKACIJA OBLIKA OSCILOVANJA U DINAMIČKOJ ANALIZI IMPELERA CENTRIFUGALNE PUMPE METODOM KONAČNIH ELEMENATA

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REZIME

PREGLEDNI RAD

Problemi vibracija struktura složenijih geometrijskih oblika, kao što su to impeleri centrifugalnih pumpi, se danas najčešće analiziraju korištenjem metode konačnih elemenata ili eksperimentalnim metodama kao što je modalna analiza. Svaka dinamička analiza započinje određivanjem vlastitih frekvencija, koje se dobiju rješavanjem problema vlastitih vrijednosti. Ako se analiza vrši sa ciljem optimizacije konstrukcije, potrebno je vlastite frekvencije klasificirati prema oblicima oscilovanja, da bi se mogla izvući određena zakonitost. U ovom radu je prikazan način klasifikacije oblika oscilovanja impelera jednog tipa centrifugalne pumpe, za koji je dokazano da se ponaša veoma slično okrugloj ploči.

Ključne riječi: metod konačnih elemenata, vibracije, oblici oscilovanja, impeler

IDENTIFICATION OF VIBRATION MODE SHAPES IN DYNAMIC ANALYSIS OF A CENTRIFUGAL PUMP IMPELLER BY USE OF FINITE ELEMENT METHOD

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SUMMARY

SUBJECT REVIEW

Problems of vibrating structures with complex geometry, such as centrifugal pump impellers, are being analysed by means of Finite Element Method or experimental methods such as modal analysis. Each dynamic analysis begins with determining natural frequencies, derived by solving eigenvalues problem. If the analysis is performed in order to optimise construction, it is necessary to classify natural frequencies according to vibration mode shapes, to enable establishment of certain dependencies. This paper shows how impeller vibration mode shapes can be classified for one type of centrifugal pump, and it proves that the impeller presented acts very similarly to circular plate.

Key Words: Finite Elements Method, Vibrations, Mode shapes, Impeller

1. UVOD

Impeler centrifugalne pumpe se ponaša slično okrugloj ploči [2], tako da se može pretpostaviti da se vlastite frekvencije impelera mogu izvesti sličnim postupkom koji se koristi za vibracije ploča. Dinamička jednačina okrugle ploče se može izvesti analitički, polazeći od Hamiltonovog principa, uz korištenje Besselovih funkcija.

1. INTRODUCTION

Centrifugal pump impeller acts similarly to circular plate [2]. Therefore, assumption can be made that impeller natural frequencies can be calculated using similar procedure as the one used in plate vibration. Dynamic equation of circular plate motion can be derived analytically, starting from Hamilton's principle, and using Bessel functions.

Međutim, za složenije geometrijske oblike, kao što je impeller centrifugalne pumpe, racionalnije je koristiti približne metode, kao što je metod konačnih elemenata. Ova metoda se zasniva na Rayleigh-Ritzovoj metodi, a svodi se na problem vlastitih vrijednosti u matricnom obliku. Numerička rješenja tog problema ne daju detaljne informacije o oblicima oscilovanja (modovima), nego daju samo vlastite frekvencije i koordinate tačaka u čvorovima konačnih elemenata. Vlastite frekvencije su date po veličini, a obuhvataju sve oblike vibracija, i poprečne i torzione. Identifikacija tih oblika zahtijeva dobro poznavanje analitičkog rješenja za slične oblike. Kod impelera jedne vrste centrifugalnih pumpi, to su okrugle ploče.

For more complex geometric shapes, however such as centrifugal pump impeller, it is more rational to use approximate methods, such as finite element method. This method is based on Rayleigh-Ritz method, and it is basically eigenvalue problem in matrix form. Numerical solutions to this problem do not give detailed information about vibration modes, but only natural frequencies and coordinates of points in finite element nodes. Natural frequencies are sorted by size, and are given for both lateral and torsional vibrations. Identification of these vibration modes requires fair knowledge on analytical solution for similar shapes. For impellers of one type of centrifugal pump, such shapes are circular plates.

2. DINAMIČKA JEDNAČINA KRETANJA OKRUGLE PLOČE

2. DYNAMIC EQUATION OF MOTION FOR CIRCULAR PLATE

Tanka ploča je dvodimenzionalna analogija Euler-Beronullijeve grede (kada su zanemarena tangencijalna naprezanja). Dvodimenzionalna analogija Timoshenkove grede (kada se uzimaju u obzir tangencijalna naprezanja i inercione sile usljed rotacije) je poznata u literaturi kao Mindlin-Timoshenkova teorija.

Thin plate is two-dimensional analogy of Euler-Beronulli beam (where tangential stresses are neglected). Two-dimensional analogy of Timoshenko beam (including both tangential stresses and inertial forces due to rotation of the plate) is known in literature as Mindlin-Timoshenko theory.

Diferencijalna jednačina slobodnih vibracija okrugle ploče u cilindričnim koordinatama glasi:

Differential equation of free vibration of circular plate in cylindric coordinates is:

$$D\Delta\Delta w - \rho h \ddot{w} = 0 \dots\dots\dots (1)$$

$$D\Delta\Delta w - \rho h \ddot{w} = 0 \dots\dots\dots (1)$$

Gdje je Δ Laplace-ov operator:

Where Δ is Laplace's operator:

$$\Delta = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \dots\dots\dots (2)$$

$$\Delta = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \dots\dots\dots (2)$$

Potrebno je odrediti uslove pod kojima će sve tačke ploče izvoditi harmonijske vibracije istom frekvencijom i istom fazom. Pretpostavi se oblik funkcije w :

The goal is to determine conditions where all points on the plate will perform harmonic vibration with same frequency and in the same phase. The shape of function w is assumed:

$$w(r, \varphi, t) = W(r, \varphi, t)e^{i\omega t} \dots\dots\dots (3)$$

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Ako se uvede konstanta c :

Introducing constant c :

$$c^2 = \frac{D}{\rho h} \dots\dots\dots (4)$$

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Jednačina (1) se može napisati u obliku:

Equation (1) can be written as:

$$c^2 \Delta\Delta w - \ddot{w} = 0 \dots\dots\dots (5)$$

$$c^2 \Delta\Delta w - \ddot{w} = 0 \dots\dots\dots (5)$$

Uvrštavanjem (3) u (4) dobije se:

Including (3) into (4):

$$c^2 \Delta\Delta W(r, \varphi) - \omega^2 W(r, \varphi) = 0 \dots\dots\dots (6)$$

$$c^2 \Delta\Delta W(r, \varphi) - \omega^2 W(r, \varphi) = 0 \dots\dots\dots (6)$$

ili

or

$$\Delta\Delta W - \lambda^4 W = (\Delta - \lambda^2)(\Delta + \lambda^2)W = 0 \dots\dots\dots (7)$$

$$\Delta\Delta W - \lambda^4 W = (\Delta - \lambda^2)(\Delta + \lambda^2)W = 0 \dots\dots\dots (7)$$

gdje je:

$$\lambda^2 = \frac{\omega}{c} \dots\dots\dots (8)$$

Rješenje jednačine $\Delta - \lambda^2 = 0$ je:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A\mathfrak{J}_m(\lambda r) + BY_m(\lambda r)] \dots\dots (9)$$

Rješenje jednačine $\Delta + \lambda^2 = 0$ je:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [C\mathfrak{J}_m(i\lambda r) + DY_m(i\lambda r)] \dots\dots (10)$$

Opšte rješenje jednačine (5) glasi:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A\mathfrak{J}_m(\lambda r) + BY_m(\lambda r) + C\mathfrak{J}_m(i\lambda r) + DY_m(i\lambda r)] \dots\dots (11)$$

gdje su $\mathfrak{J}_m(\lambda r)$ i $\mathfrak{J}_m(i\lambda r)$ Besselove funkcije prve vrste, a $Y_m(\lambda r)$ i $Y_m(i\lambda r)$ Besselove funkcije druge vrste.

Kako se radi o punoj ploči, može se staviti da je $B = D = 0$, jer funkcije $Y_m(\lambda r)$ i $Y_m(i\lambda r)$ imaju u centru ploče singularitete. Argument funkcije $\mathfrak{J}_m(i\lambda r)$ je imaginaran broj. Uvodi se oznaka

$$I_m(z) = i^{-m} \mathfrak{J}_m(iz)$$

gdje funkcija $I_m(z)$ ima sljedeće osobine:

$$I_{m-1}(z) - I_{m+1}(z) = \frac{2m}{z} I_m(z)$$

$$\frac{d}{dz} I_m(z) = \frac{1}{2} [I_{m-1}(z) + I_{m+1}(z)] \dots\dots\dots (12)$$

$$\int I_0(z) dz = zI_1(z)$$

$$\int I_1(z) dz = I_0(z)$$

Prema (12), za punu kružnu ploču dopuštena su sljedeća rješenja:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A_1\mathfrak{J}_m(\lambda r) + B_1I_m(\lambda r)] \dots\dots (13)$$

Za slobodno oslonjenu ploču (vanjski poluprečnik $r = R$) važe granični uslovi:

where:

$$\lambda^2 = \frac{\omega}{c} \dots\dots\dots (8)$$

Solution of equation $\Delta - \lambda^2 = 0$ is:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A\mathfrak{J}_m(\lambda r) + BY_m(\lambda r)] \dots\dots (9)$$

Solution of equation $\Delta + \lambda^2 = 0$ is:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [C\mathfrak{J}_m(i\lambda r) + DY_m(i\lambda r)] \dots\dots (10)$$

General solution of equation (5) is:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A\mathfrak{J}_m(\lambda r) + BY_m(\lambda r) + C\mathfrak{J}_m(i\lambda r) + DY_m(i\lambda r)] \dots\dots (11)$$

where $\mathfrak{J}_m(\lambda r)$ and $\mathfrak{J}_m(i\lambda r)$ are first order Bessel functions, and $Y_m(\lambda r)$ and $Y_m(i\lambda r)$ second order Bessel functions.

As this is solid plate, it can be assumed that $B = D = 0$, since $Y_m(\lambda r)$ and $Y_m(i\lambda r)$ have singularities in the centre of the plate. Argument of function $\mathfrak{J}_m(i\lambda r)$ is imaginary number.

$$I_m(z) = i^{-m} \mathfrak{J}_m(iz)$$

where function $I_m(z)$ has these characteristics:

$$I_{m-1}(z) - I_{m+1}(z) = \frac{2m}{z} I_m(z)$$

$$\frac{d}{dz} I_m(z) = \frac{1}{2} [I_{m-1}(z) + I_{m+1}(z)] \dots\dots\dots (12)$$

$$\int I_0(z) dz = zI_1(z)$$

$$\int I_1(z) dz = I_0(z)$$

According to (12), for solid circular plate following solutions are permissible:

$$W(r, \varphi) = \frac{\cos}{\sin} (m\varphi) [A_1\mathfrak{J}_m(\lambda r) + B_1I_m(\lambda r)] \dots\dots (13)$$

For pinned plate (external radius $r = R$) following boundary conditions apply:

$$W(R, \varphi) = 0$$

$$\Delta W(R, \varphi) = 0; \quad \dots\dots\dots (14)$$

$$\left[\frac{\partial^2 W}{\partial r^2} + v \left(\frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \varphi^2} \right) \right]_{r=R} = 0$$

Uvrštavanjem (13) u (14), dobiju se dvije homogene jednačine čijim se rješavanjem dolazi do frekventne jednačine.

$$W(R, \varphi) = 0$$

$$\Delta W(R, \varphi) = 0; \quad \dots\dots\dots (14)$$

$$\left[\frac{\partial^2 W}{\partial r^2} + v \left(\frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \varphi^2} \right) \right]_{r=R} = 0$$

Introducing (13) into (14), two homogenous equations are derived, and their solution leads to frequency equation.

3. OBLICI OSCILOVANJA OKRUGLIH PLOČA

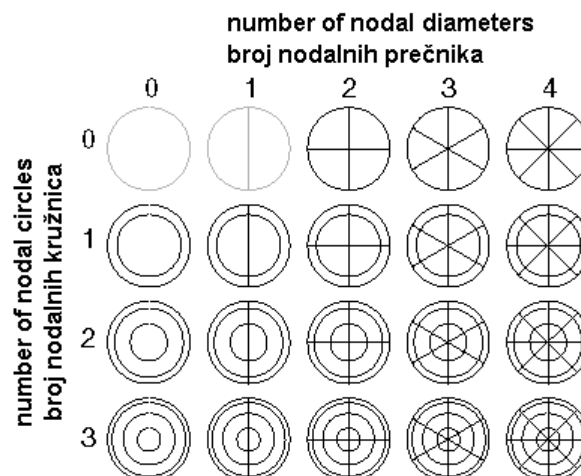
Oblici oscilovanja (modovi, tonovi) okruglih ploča se mogu prikazati pomoću nodalnih linija. To su geometrijska mjesta tačaka koje se ne pomjeraju za određeni oblik oscilovanja.

Površine na koje se ploča dijeli nodalnim linijama se kreću u suprotnim smjerovima, čime se definiše oblik oscilovanja ploče.

3. MODE SHAPES OF CIRCULAR PLATES

Vibration mode shapes of circular plates can be represented using nodal lines. Nodal lines consist of stationary points, i.e. points that do not move for specific mode shape.

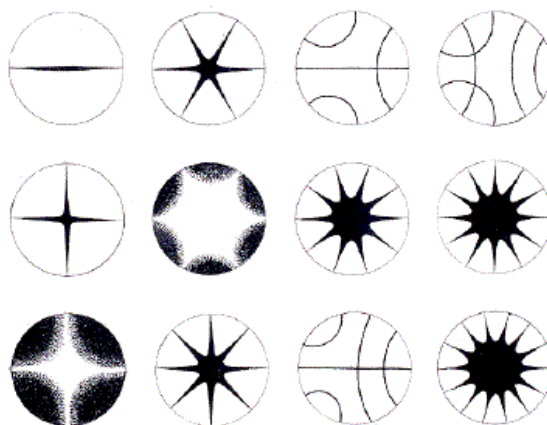
Nodal lines divide the plate into areas moving in opposite directions, thus defining vibration mode shape.



Slika 1. Nodalne linije i nodalne kružnice
Figure 1. Nodal lines and nodal circles

Njemački fizičar Ernst Chladni je 1787. godine odredio oblike tih linija eksperimentalno, sipanjem pijeska na vibrirajuće ploče (za pobudu vibracija koristio je violinsko gudačo). Pijesak se zadržavao samo na mjestima na kojima je amplituda vibracija bila najmanja, odnosno na nodalnim linijama. Švicarski fizičar Walter Ritz je 1904. razvio analitički metod da bi reprodukovao Chladnijeve eksperimente.

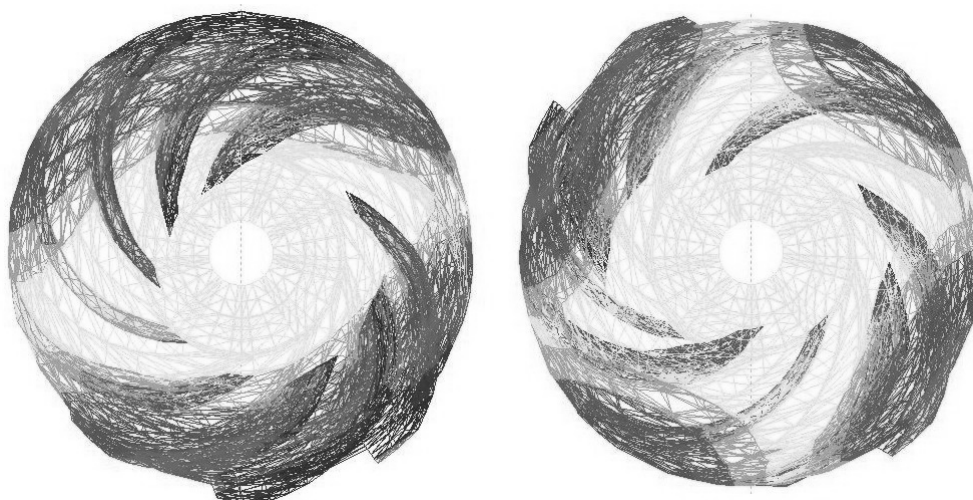
In 1787 German physicist Ernst Chladni determined the shapes of nodal lines experimentally, pouring sand on vibrating plate (using violin bow to produce vibrations). The sand formed patterns remaining only on places where vibration amplitude was smallest – on nodal lines. In 1904, Swiss physicist Walter Ritz developed analytical method to reproduce Chladni's experiments.



Slika 2. Chladnijeve figure – oblici oscilovanja okrugle ploče
 Figure 2. Chladni figures – circular plate mode shapes

Isti eksperiment je 1967. ponovio švicarski naučnik Hans Jenny, ali je ovaj put za pobudu vibracija koristio uređaj vlastite izrade pod nazivom "tonoskop" [1]. To je bio kristalni oscilator pomoću kojeg su se frekvencija i amplituda vibracija dale kontrolisati. Potvrdio je rezultate koje je dobio Chladni, i proširio ih je na ploče sa pijeskom, željeznim prahom, srebrom, polenom, gasovima, vodom, plastičnim supstancama i viskoznim tečnostima. Došao je do zaključka da fluid (voda) prijanja na bijele površine Chladnijevih figura, a da se na crnim površinama (nodalne linije) fluid odvaja od površine ploče. Takođe je zaključio da samo frekvencija utiče na oblik figura (broj nodalnih linija), a da amplituda utiče samo na intenzitet turbulencija i brzinu kojom se materijal kreće po ploči.

In 1967, Swiss scientist Hans Jenny performed the same experiment. He constructed special device "Tonoscope" [1] to set the plates vibrating instead of violin bow. It was crystal oscillator with adjustable frequency and amplitude. He confirmed Chladni's results, and he expanded these results to plates with sand, iron filings, silver, spores, gases, water, plastic substances and viscous liquids. He came to conclusion that fluid (water) remains attached to white (vibrating) areas of Chladni figures, and it separates from black areas (non-vibrating nodal lines). He also concluded that the shape of these patterns (number of nodal lines) depends only on frequency, and amplitude depends on intensity of turbulence and velocity by which material moves on the plate.



Slika 3. Chladnijeve figure nastale vibracijom impelera centrifugalne pumpe
 Figure 3. Chladni figures formed by vibration centrifugal pump impeller

4. ODREĐIVANJE VLASTITIH FREKVENCIJA METODOM KONAČNIH ELEMENATA

Za određivanje vlastitih frekvencija i oblika oscilovanja za slučaj slobodnih vibracija pomoću MKE polazi se od matricne jednačine kretanja diskretizirane strukture sa n stepeni slobode kretanja:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \dots\dots\dots (15)$$

gdje su:

[M] – matrica masa

[C] – matrica prigušenja

[K] – matrica krutosti

$\{\ddot{q}\}, \{\dot{q}\}, \{q\}$ - vektori generalisanih ubrzanja, brzina i pomjeranja

Rješenja jednačine (15) se pretpostave u obliku:

$$\{q\} = \{u\}e^{\lambda t} \dots\dots\dots (16)$$

tako da se problem (15) svodi na problem vlastitih vrijednosti:

$$(\lambda^2 [M] + \lambda [C] + [K])\{u\} = \{0\} \dots\dots\dots (17)$$

odnosno

$$[H(\lambda)]_{n \times n} \{u\} = \{0\} \dots\dots\dots (18)$$

$$\{v\}^T [H(\lambda)] = \{0\}^T$$

Rješavanjem (18) dobivaju se vlastite vrijednosti λ_i i vlastiti vektori $\{u_i\}, \{v_i\}$, za $i=1,2,\dots,2n$. To znači da za matricu dinamičke krutosti $[H(\lambda)]_{n \times n}$ postoji $2n$ vlastitih rješenja.

Problemi (18) su problemi drugog reda po λ .

Za slučaj neprigušenih slobodnih vibracija pretpostavlja se rješenje u obliku:

$$\{q(t)\} = \{u\}e^{i\omega t} \dots\dots\dots (19)$$

pa se dobija:

$$[[K] - \omega^2 [M]]\{u\}e^{i\omega t} = \{0\} \dots\dots\dots (20)$$

odnosno:

$$[[K] - \omega^2 [M]]\{u\} = \{0\} \dots\dots\dots (21)$$

Da rješenje ne bi bilo trivijalno, mora biti determinanta matrice:

$$\det[[K] - \omega^2 [M]] = 0 \dots\dots\dots (22)$$

Iz ove algebarske jednačine dobija se niz od n pozitivnih realnih vrijednosti ω^2 koje predstavljaju vlastite vrijednosti datog problema. Vrijednosti $\omega_i, i=1,2,\dots,n$ predstavljaju neprigušene vlastite frekvencije sistema. Kada se te vrijednosti uvrste u (21), dobije se n vektorskih rješenja koja predstavljaju oblike oscilovanja sistema (modeve).

4. CALCULATION OF NATURAL FREQUENCIES USING FINITE ELEMENT METHOD

Process of determination of natural frequencies and mode shapes for free vibration using Finite Element Method starts from matrix equation of motion of discretized structure with n degrees of freedom.

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \dots\dots\dots (15)$$

where:

[M] – mass matrix

[C] – damping matrix

[K] – stiffness matrix

$\{\ddot{q}\}, \{\dot{q}\}, \{q\}$ - vectors of generalised acceleration, velocity and displacement

Solutions of (15) is assumed in following form:

$$\{q\} = \{u\}e^{\lambda t} \dots\dots\dots (16)$$

Therefore, the problem (15) is reduced to eigenvalue problem:

$$(\lambda^2 [M] + \lambda [C] + [K])\{u\} = \{0\} \dots\dots\dots (17)$$

or

$$[H(\lambda)]_{n \times n} \{u\} = \{0\} \dots\dots\dots (18)$$

$$\{v\}^T [H(\lambda)] = \{0\}^T$$

Solution of (18) gives eigenvalues λ_i and eigenvectors $\{u_i\}, \{v_i\}$, for $i=1,2,\dots,2n$. It means that for dynamic stiffness matrix $[H(\lambda)]_{n \times n}$ there are $2n$ eigensolutions.

Problems (18) are second order problems by λ .

In the case of undamped free vibration, it is assumed that solution has the following form:

$$\{q(t)\} = \{u\}e^{i\omega t} \dots\dots\dots (19)$$

which brings:

$$[[K] - \omega^2 [M]]\{u\}e^{i\omega t} = \{0\} \dots\dots\dots (20)$$

or:

$$[[K] - \omega^2 [M]]\{u\} = \{0\} \dots\dots\dots (21)$$

To have non-trivial solution, determinant of the matrix should be:

$$\det[[K] - \omega^2 [M]] = 0 \dots\dots\dots (22)$$

This algebraic equation gives array of n positive real values ω^2 that represent eigenvalues of given problem. Values $\omega_i, i=1,2,\dots,n$ represent undamped natural frequencies of the system. When these values are entered into (21), n vector solutions are derived, which represent vibration modes of the given system.

5. KLASIFIKACIJA VLASTITIH FREKVENCIJA DOBIJENIH METODOM KONAČNIH ELEMENATA KORIŠTENJEM CHLADNIJEVIH FIGURA

U tabeli 1 su prikazani rezultati proračuna vlastitih frekvencija kako ih daje software koji se zasniva na metodi konačnih elemenata. Rezultati su sortirani samo prema veličini, i nema razlike između torzionih i poprečnih vibracija. Izračunato je prvih 10 vlastitih frekvencija, od kojih se neke ponavljaju.

Na slici 4.a. je prikazan dijagram koji daje lažnu sliku o zavisnosti debljine diska i vlastitih frekvencija, jer u sebi ne uključuje klasifikaciju vibracija prema oblicima oscilovanja.

Na slici 4.b. prikazan je dijagram na osnovu rezultata sortiranih prema oblicima oscilovanja (desni dio tabele 1). Vlastite frekvencije nisu označene rednim brojevima, nego imaju dva indeksa. Prvi indeks predstavlja broj nodalnih prečnika, a drugi broj nodalnih kružnica. Indeks "00" predstavlja torzione vibracije.

5. CLASSIFICATION OF NATURAL FREQUENCIES DERIVED USING FINITE ELEMENT METHOD BY MEANS OF CHLADNI FIGURES

Table 1 shows results of calculated natural frequencies as they are given by finite element method software. These results are sorted only by size, and there is no distinction between torsional and lateral vibrations. First 10 natural frequencies are calculated, and some values are repeated.

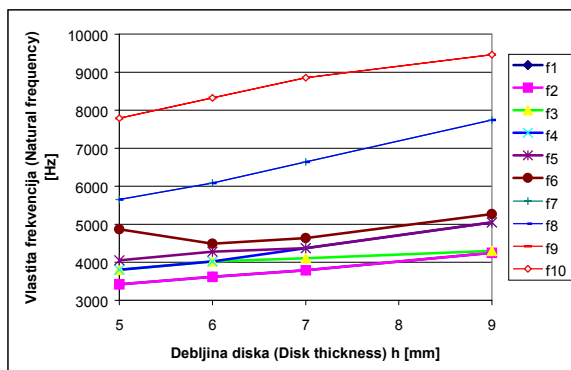
Figure 4.a. shows diagram with false dependence between disk thickness and natural frequencies, since it does not involve vibration classification according to vibration modes.

Figure 4.b. shows diagram according to results sorted by vibration modes (right-hand half of the table 1). Natural frequencies are not denominated with ordinal numbers, but they have two indexes. First index represents the number of nodal diameters, and the second one, the number of nodal circles. Index "00" represents torsional vibration.

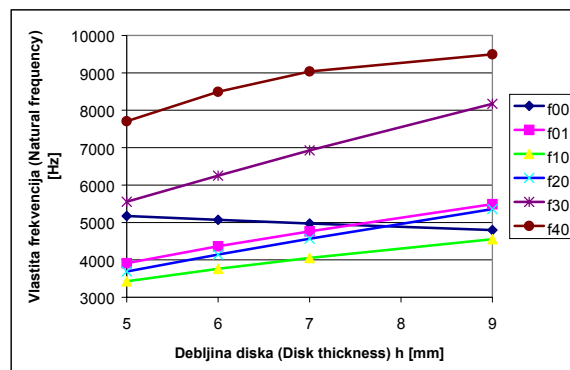
Tabela 1. Vlastite frekvencije izračunate metodom konačnih elemenata

Table 1. Natural frequencies calculated using finite element method

Vlastite frekvencije sortirane po veličini Natural frequencies sorted by size					Vlastite frekvencije sortirane po obliku oscilovanja Natural frequencies sorted by vibration modes				
Debljina diska (Disk thickness)	5 mm	6 mm	7 mm	9 mm	Debljina diska (Disk thickness)	5 mm	6 mm	7 mm	9 mm
f_1	3422	3619	3793	4249	f_{00}	4870	4488	4106	4301
f_2	3422	3619	3793	4249	f_{01}	4052	4279	4636	5270
f_3	3806	4021	4106	4301	f_{10}	3422	3619	3793	4249
f_4	3806	4021	4372	5051	f_{20}	3806	4021	4372	5051
f_5	4052	4279	4372	5051	f_{30}	5652	6086	6642	7742
f_6	4870	4488	4636	5270	f_{40}	7792	8324	8856	9461
f_7	5652	6086	6642	7742					
f_8	5652	6086	6642	7742					
f_9	7792	8324	8856	9461					
f_{10}	7792	8324	8856	9461					

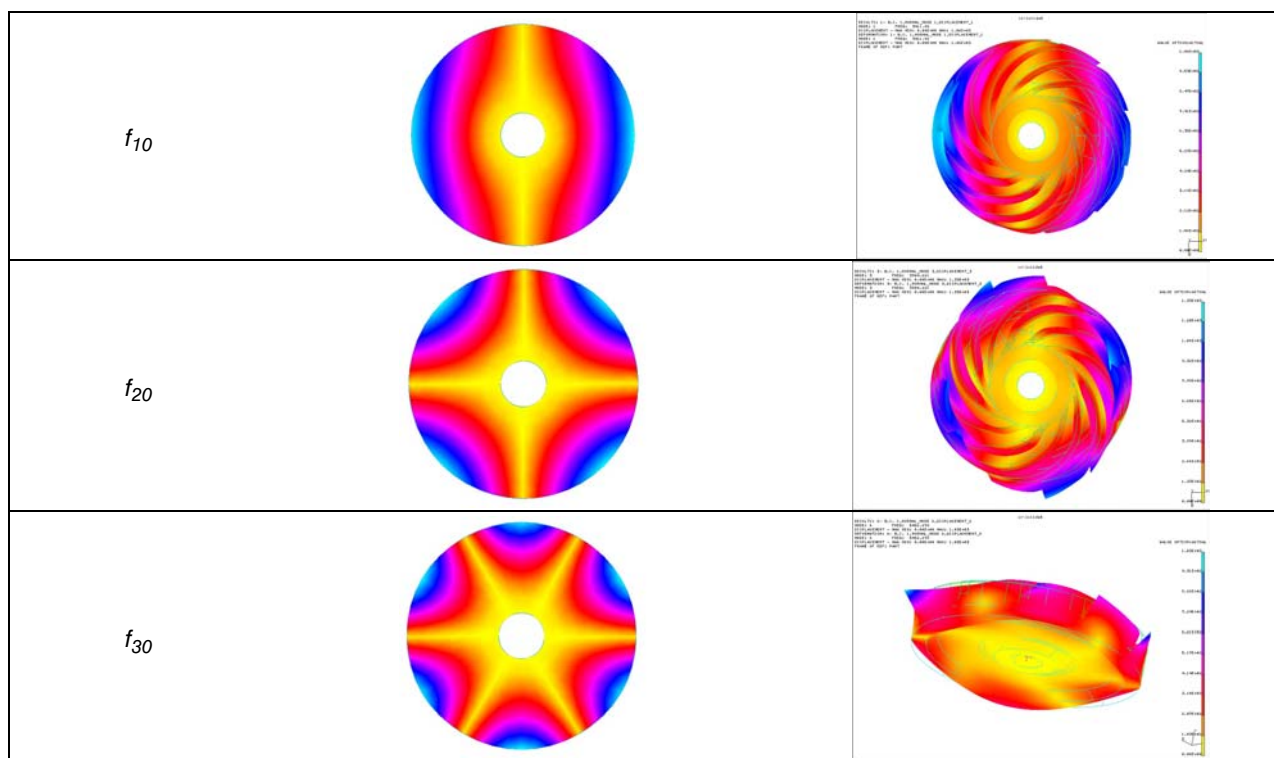


a) Sortirane po veličini (Sorted by size)



b) Sortirane prema oblicima oscilovanja (Sorted by vibration modes)

Slika 4. Vlastite frekvencije izračunate metodom konačnih elemenata
Figure 4. Natural frequencies calculated using finite element method



Slika 5. Oblici oscilovanja okrugle ploče i impelera centrifugalne pumpe
 Figure 5. Vibration mode shapes of circular plate and centrifugal pump impeler

6. ZAKLJUČAK

Metoda konačnih elemenata daje dobre rezultate u proračunu vlastitih frekvencija. Međutim, ako se želi izvući zavisnost nekih parametara, treba poznavati i fizikalni karakter vibracija strukture koja se analizira. Chladnijeve figure mogu biti veoma korisne kao polazna osnova za identifikaciju i klasifikaciju oblika oscilovanja struktura pločastog oblika, kao što su impeleri nekih vrsta centrifugalnih pumpi, željeznički točkovi, ploče računarskih tvrdih diskova, itd.

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6. CONCLUSION

Finite element method gives fair results in calculation of natural frequencies. Nevertheless, if dependence of some parameters is needed, it is necessary to be familiar with physical character of vibrations of the structure being analysed. Chladni figures can be of great importance as a basis for identification and classification of vibration modes of the plate-shaped structures, such as impellers of some centrifugal pump, rail wheels, plates in computer hard disks, etc.

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